Structural optimisation of wind turbine towers based on finite element analysis and genetic algorithm

Lin Wang1*, Athanasios Kolios1, Maria Martinez Luengo1, Xiongwei Liu2

1Centre for Offshore Renewable Energy Engineering, School of Water, Energy and Environment, Cranfield University, Cranfield, MK43 0AL, UK
2Entrust Microgrid, Lancaster Environment Centre, Gordon Manley Building, Lancaster University, LA1 4YQ, UK

Abstract

A wind turbine tower supports the main components of the wind turbine (e.g. rotor, nacelle, drive train components, etc.). The structural properties of the tower (such as stiffness and natural frequency) can significantly affect the performance of the wind turbine and the cost of the tower is a considerable portion of the overall wind turbine cost. Therefore, an optimal structural design of the tower, which has a minimum cost and meets all design criteria (such as stiffness and strength requirements), is crucial to ensure efficient, safe and economic design of the whole wind turbine system. In this work, a structural optimisation model for wind turbine towers has been developed based on a combined parametric FEA (finite element analysis) and GA (genetic algorithm) model. The top diameter, bottom diameter and thickness distributions of the tower are taken as design variables. The optimisation model minimises the tower mass with six constraint conditions, i.e. deformation, ultimate stress, fatigue, buckling, vibration and design variable constraints. After validation, the model has been applied to the structural optimisation of a 5MW wind turbine tower. The results demonstrate that the proposed structural optimisation model is capable of accurately and effectively achieving an optimal structural design of wind turbine towers, which significantly improves the efficiency of structural optimisation of wind turbine towers. The developed framework is generic in nature and can be employed for a series of related problems, when advanced numerical models are required to predict structural responses and to optimise the structure.

1. Introduction

Wind power is capable of providing a competitive solution to battle the global climate change and energy crisis, making it the most promising renewable energy resource. As an abundant and inexhaustible energy resource, wind power is available and deployable in many regions of the world. Therefore, regions such as Northern Europe and China are making considerable efforts in exploring wind power resources. According to Global Wind Energy Council (GWE, 2016), the global wind power cumulative capacity reached 432 GW at the end of 2015, growing by 62.7 GW over the previous year. It is predicted that wind power could reach a total installed global capacity of 2,000 GW by 2030, supplying around 19% of global electricity (Council, 2015).

* Corresponding author. Tel.: +44(0)1234754706; E-mail address: lin.wang@cranfield.ac.uk
A wind turbine tower supports the main components of the wind turbine (e.g. rotor, nacelle, drive train components, etc.) and elevates the rotating blades at a certain elevation to obtain desirable wind characteristics. The structural properties of a wind turbine tower, such as the tower stiffness and natural frequency, can significantly affect the performance and structural response of the wind turbine, providing adequate strength to support induced loads and avoiding resonance. Additionally, the cost of the tower is a significant portion of the overall wind turbine cost (Aso and Cheung, 2015). Therefore, an optimal structural design of the tower, which has a minimum cost and meets all design criteria (such as stiffness and strength requirements), is crucial to ensure efficient, safe and economic design of the whole wind turbine system. It also contributes to reducing the cost of energy, which is one of the long-term research challenges in wind energy (van Kuik et al., 2016).

The structural optimisation model of a wind turbine tower generally consists of two components, i.e. 1) a wind turbine tower structural model, which analyses the structural performance of the tower, such as tower mass and deformations; and 2) an optimisation algorithm, which deals with design variables and searches for optimal solutions.

Structural models used for wind turbine towers can be roughly classified into two groups, i.e. 1D (one-dimensional) beam model and 3D (three-dimensional) FEA (finite element analysis) model. The 1D beam model discretises the tower into a series of beam elements, which are characterised by cross-sectional properties (such as mass per unit length and cross-sectional stiffness). Due to its efficiency and reasonable accuracy, the 1D beam model has been widely used for structural modelling of wind turbine towers (Zhao and Maissner, 2006, Murtagh et al., 2004) and blades (Wang et al., 2014b, Wang et al., 2014a, Wang, 2015). Although it is efficient, the beam model is incapable of providing some important information for the tower design, such as detailed stress distributions within the tower structure, hence making such models incapable of capturing localised phenomena such as fatigue. In order to obtain the detailed information, it is necessary to construct the tower structure using 3D FEA. In 3D FEA, wind turbine towers are generally constructed using 3D shell or brick elements. Compared to the 1D beam model, the 3D FEA model provides more accurate results and is capable of examining detailed stress distributions within the tower structure. Due to its high fidelity, the 3D FEA model has been widely used for modelling wind turbine structures (Wang et al., 2015, Wang et al., 2016b, Stavridou et al., 2015). Therefore, the 3D FEA model is chosen in this study to model the wind turbine tower structure.

Optimisation algorithms can be roughly categorised into three groups (Herbert-Acero et al., 2014), i.e. exact algorithms, heuristic algorithms and metaheuristic algorithms. Exact algorithms, which find the best solution by evaluating every possible combination of design variables, are very precise because all possible combinations are evaluated. However, they become time-consuming and even infeasible when the number of design variables is large, requiring huge computational resources to evaluate all possible combinations. Heuristic algorithms, which find near-optimal solutions based on semi-empirical rules, are more efficient.
than exact algorithms. However, they are problem-dependent and their accuracy highly depends on the accuracy of semi-empirical rules, limiting their applications to some extent. Metaheuristic algorithms, which are more complex and intelligent heuristics, are high-level problem-independent algorithms to find near-optimal solutions. They are more efficient than common heuristic algorithms and are commonly based on optimisation processes observed in the nature, such as PSO (particle swarm optimisation) (Kennedy, 2011), SA (simulated annealing) (Dowsland and Thompson, 2012) and GA (genetic algorithm) (Sivanandam and Deepa, 2007). Among these metaheuristic algorithms, the GA, which searches for the optimal solution using techniques inspired by genetics and natural evolution, is capable of handling a large number of design variables and avoiding being trapped in local optima, making it the most widely used metaheuristic algorithm (Wang et al., 2016a). Therefore, the GA is selected in this study to handle the design variables and to find the optimal solution.

This paper attempts to combine FEA and GA to develop a structural optimisation model for onshore wind turbine towers. A parametric FEA model of wind turbine towers is developed and validated, and then coupled with GA to develop a structural optimisation model. The structural optimisation model is applied to a 5MW onshore wind turbine to optimise the 80m-height tower structure.

This paper is structured as follows. Section 2 presents the parametric FEA model of wind turbine towers. Section 3 presents the GA model. Section 4 presents the optimisation model by combining the parametric FEA model and GA model. Results and discussions are provided in Section 5, followed by conclusions in Section 6.

2. Parametric finite element analysis (FEA) model of wind turbine towers

2.1. Model description

A parametric FEA model of wind turbine towers is established using ANSYS, which is a widely used commercial FE software. The parametric FEA model enables the design parameters of wind turbine towers to be easily modified to create various tower models. The flowchart of the parametric model of wind turbine towers is presented in Fig. 1.
Figure 1. Flowchart of the parametric FEA model for wind turbine towers

Each step of the flowchart Fig. 1 is detailed below.

1) Define design parameters: In the first step, design parameters of the wind turbine towers, such as tower top and bottom diameters, are defined.

2) Create tower geometry: The tower geometry is created based on the bottom-up approach, which creates low dimensional entities (such as lines) first and then creates higher dimensional entities (such as areas) on top of low dimensional entities.

3) Define and assign material properties: In this step, material properties (such as Young’s modulus and Poisson’s ratio) are defined and then assigned to the tower structure.

4) Define element type and generate mesh: Due to the fact that wind turbine towers are generally thin-wall structures, they can be effectively and accurately modelled using shell elements. The element type used here is the shell element Shell281, which has eight nodes with six degrees of freedom at each node and it is well-suited for linear, large rotation, and/or large strain nonlinear applications. Additionally, a regular quadrilateral mesh generation method is used to generate high quality element, ensuring the computational accuracy and saving on computational time.

5) Define boundary conditions: In this step, boundary conditions are applied. The types of boundary conditions are dependent on the types of analyses. For instance, a fixed boundary condition is applied to the tower bottom for modal analysis.

6) Solve and post-process: Having defined design parameters, geometry, materials, element types, mesh and boundary conditions, a variety of analyses (such as static analysis, modal analysis and buckling analysis) can be performed. The simulation results, such as tower deformations and stress distributions, are then plotted using post-processing functions of ANSYS software.

2.2. Validation of the parametric FEA model

A case study is performed to validate the parametric FEA model of wind turbine towers. The NREL 5MW wind turbine (Jonkman et al., 2009), which is a representative of large-scale of HAWTs is chosen as an
example. The NREL 5MW wind turbine is a reference wind turbine designed by NREL (National Renewable Energy Laboratory), and it is a conventional three-bladed upwind HAWT, utilising variable-speed variable-pitch control. The geometric and material properties of NREL 5MW wind turbine tower are presented in Table 1. The steel density is increased from a typical value of 7,850 kg/m$^3$ to a value of 8,500 kg/m$^3$ to take account of paint, bolts, welds and flanges that are not accounted for in the tower thickness data (Jonkman et al., 2009). The diameters and thickness of the tower are linearly tapered from the tower base to tower top.

Table 1. Geometric and material properties of the NREL 5MW wind turbine tower (Jonkman et al., 2009)

<table>
<thead>
<tr>
<th>Properties</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tower height [m]</td>
<td>87.6</td>
</tr>
<tr>
<td>Tower top outer diameter [m]</td>
<td>3.87</td>
</tr>
<tr>
<td>Tower top wall thickness</td>
<td>0.0247</td>
</tr>
<tr>
<td>Tower base outer diameter [m]</td>
<td>6</td>
</tr>
<tr>
<td>Tower base wall thickness [m]</td>
<td>0.0351</td>
</tr>
<tr>
<td>Density [kg/m$^3$]</td>
<td>8500</td>
</tr>
<tr>
<td>Young’s modulus [GPa]</td>
<td>210</td>
</tr>
<tr>
<td>Shear modulus [GPa]</td>
<td>80.8</td>
</tr>
</tbody>
</table>

The parametric FEA model presented in Section 2.1 is applied to the modal analysis of the NREL 5MW wind turbine tower. In this case, the tower is fixed at the tower bottom and free-vibration (no loads on the tower), and tower head mass is ignored. A regular quadrilateral mesh generation method is used to generate high quality elements. In order to determine the appropriate mesh size, a mesh sensitivity study is carried out for the first 6 modal frequencies, of which the results are presented in Table 2. As can be seen from Table 2, the modal frequencies converge at a mesh size of 0.5m, with a maximum relative difference (0.002%) occurring for the $2^{nd}$ side-to-side mode when compared to further mesh refinement with a mesh size of 0.25m. Therefore, 0.5m is deemed as the appropriate element size. The created mesh is presented in Fig. 2, and the total number of element is 6,960.

Table 2. FEA mesh sensitivity analysis

<table>
<thead>
<tr>
<th>Modal frequencies</th>
<th>2m sizing</th>
<th>1m sizing</th>
<th>0.5m sizing</th>
<th>0.25m sizing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^{st}$ SS (Hz)</td>
<td>0.8781</td>
<td>0.8782</td>
<td>0.8782</td>
<td>0.8782</td>
</tr>
<tr>
<td>$1^{st}$ FA (Hz)</td>
<td>0.8855</td>
<td>0.8855</td>
<td>0.8856</td>
<td>0.8856</td>
</tr>
<tr>
<td>$2^{nd}$ SS (Hz)</td>
<td>4.2315</td>
<td>4.2305</td>
<td>4.2276</td>
<td>4.2275</td>
</tr>
<tr>
<td>$2^{nd}$ FA (Hz)</td>
<td>4.2463</td>
<td>4.2469</td>
<td>4.2429</td>
<td>4.2428</td>
</tr>
</tbody>
</table>

(where SS refers to side-to-side; FA refers to force-aft)
Table 3 compare the results from the present FEA model against the results from ADAMS software reported in Ref. (Jonkman and Bir, 2010).

<table>
<thead>
<tr>
<th>Mode frequencies</th>
<th>ADAMS (Jonkman and Bir, 2010)</th>
<th>Present FEA model</th>
<th>%Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st SS (Hz)</td>
<td>0.8904</td>
<td>0.8782</td>
<td>1.37</td>
</tr>
<tr>
<td>1st FA (Hz)</td>
<td>0.8904</td>
<td>0.8856</td>
<td>0.54</td>
</tr>
<tr>
<td>2nd SS (Hz)</td>
<td>4.3437</td>
<td>4.2276</td>
<td>2.67</td>
</tr>
<tr>
<td>2nd FA (Hz)</td>
<td>4.3435</td>
<td>4.2429</td>
<td>2.32</td>
</tr>
</tbody>
</table>

As can be seen from Table 3, the force-aft (FA) and side-to-side (SS) tower modal frequencies calculated from the present FEA model match well with the results reported in Ref. (Jonkman and Bir, 2010), with the maximum percentage difference (2.67%) occurring for the 2nd SS mode. This confirms the validity of the present parametric FEA model of wind turbine towers.

2.3. Application of parametric FEA model to a 5MW wind turbine tower

The parametric FEA model is applied to FEA modelling of a 5MW wind turbine tower. The geometry and material properties, mesh, boundary conditions used in the FEA modelling are presented below.

2.3.1. Geometry and material properties

The geometric and material properties of 5MW wind turbine tower are presented in Table 4. Again, the steel density is increased from a typical value of 7,850 kg/m³ to a value of 8,500 kg/m³, taking account of paint, bolts, welds and flanges that are not accounted for in the tower thickness data. The tower height is 80m, and other geometric information (i.e. tower top diameter, tower bottom diameter and tower thickness
distributions) are unknown and to be determined in this study. The 3D geometric model of the tower is presented in Fig. 3.

Table 4. Geometric and material properties of the 5MW wind turbine tower

<table>
<thead>
<tr>
<th>Properties</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tower height [m]</td>
<td>80</td>
</tr>
<tr>
<td>Density [kg/m³]</td>
<td>8500</td>
</tr>
<tr>
<td>Young’s modulus [GPa]</td>
<td>210</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Figure 3. 3D geometry model of the 5MW wind turbine tower

2.3.2. Mesh

The tower structure is meshed using structured mesh with shell elements. The element size is 0.5m, which is based on the mesh sensitivity study results presented in Table 2 of Section 2.2. The mesh of the tower is presented in Fig. 4.

Figure 4. Mesh of the 5MW wind turbine tower
2.3.3. Loads and Boundary conditions

2.3.3.1. Loads

The loads on the tower arise from three sources, i.e. 1) gravity loads; 2) aerodynamic loads on the rotor; 3) wind loads on the tower itself, which are discussed below.

- **Gravity loads**

The gravity loads due to the mass of the components on the tower (such as the rotor and nacelle) and the mass of the tower itself can significantly contribute to the compression loads on the tower structure. These loads are usually taken into account by applying a point mass on the tower top.

- **Aerodynamic loads on the rotor**

The aerodynamic loads on the rotor are transferable to the loads on the tower top. For example, the thrust force on the rotor, \( T \), under a 50-year extreme wind condition with parked rotor is given by:

\[
T = \frac{1}{2} \rho V_{e}^{2} C_{T} \pi R^{2}
\]

(1)

where \( \rho \) is air density with a typical value of 1.225 kg/m\(^3\), \( V_{e} \) is the 50-year extreme wind speed, \( C_{T} \) is the thrust coefficient, and \( R \) is the rotor radius.

- **Wind loads on the tower itself**

The wind load on the tower itself is given by:

\[
F_{z} = \frac{1}{2} \rho V(z)^{2} C_{z} D(z)
\]

(2)

where \( \rho \) is the distributed wind load along the tower height per unit length; \( V(z) \) is the wind velocity at height \( z \); \( C_{z} \) is the drag coefficient for circular cross section, with a suggested value of 0.7 from IEC 61400-1 (Commission, 2005); \( D(z) \) is the external diameter at height \( z \) as the tower is tapered.

Due to wind shear, the wind velocity is varied along the tower height. \( V(z) \) in Eq. (2) can be determined by using the wind profile power law relationship:

\[
V(z) = V_{hub} \left( \frac{z}{z_{hub}} \right)^{\alpha}
\]

(3)

where \( V_{hub} \) is the wind velocity at hub height; \( z \) and \( z_{hub} \) are the height above ground and hub height, respectively; \( \alpha \) is the power law exponent with a typical value of 0.2.
2.3.3.2. Load cases

Design standard IEC61400-1 (IEC, 2005) defines twenty-two load cases for the structural design of wind turbines, covering all the operation conditions of a wind turbine, such as start up, normal operation, shut down and extreme wind condition. The types of analyses of the twenty-two load cases can be categorised into two groups, i.e. ultimate and fatigue. For simplicity, the typical load case used in the structural design of wind turbines is the ultimate load under 50-year wind condition (Cox and Echtermeyer, 2012, Bir, 2001) and fatigue load (Schubel and Crossley, 2012).

In this study, both ultimate and fatigue load cases are considered. For the ultimate load case, the 50-year extreme wind condition represents a severe load and therefore is taken as a critical load case. For the fatigue load case, wind fatigue loads for the normal operation of wind turbines are considered. Table 5 presents the static ultimate loads under extreme 50-year extreme wind condition, and Table 6 lists the fatigue loads. In this study, the two most significant components (i.e. thrust force $F_x$ and bending moment $M_y$) among the 6 components of force $F$ and moment $M$ are considered. Both ultimate and fatigue loads are taken from Ref. (LaNier, 2005) for WindPACT 5MW wind turbine, which is a reference wind turbine designed by NREL (National Renewable Energy Laboratory). The fatigue loads in Table 6 were derived through the DEL (Damage Equivalent Load) method, developed by NREL and detailed in Ref. (Freebury and Musial, 2000). It should be noted that the loads from Ref. (LaNier, 2005) are unfactored. In this study, load safety factors for ultimate aerodynamic loads and fatigue loads are respectively taken as 1.35 and 1.00, according to IEC 61400-1 (Commission, 2005). Factored values of ultimate aerodynamic loads taking account of a load safety factor of 1.35 are also presented in Table 5.

### Table 5. Ultimate loads under 50-year extreme wind condition

<table>
<thead>
<tr>
<th>Items</th>
<th>Unfactored aerodynamic loads (LaNier, 2005)</th>
<th>Factored aerodynamic loads (safety factor of 1.35)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_x$ (kN)</td>
<td>578</td>
<td>780</td>
</tr>
<tr>
<td>$M_y$ (kN-m)</td>
<td>28,568</td>
<td>38,567</td>
</tr>
</tbody>
</table>

### Table 6. Fatigue load (LaNier, 2005)

<table>
<thead>
<tr>
<th>Item</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_x$ (kN)</td>
<td>197</td>
</tr>
<tr>
<td>$M_y$ (kN-m)</td>
<td>3,687</td>
</tr>
</tbody>
</table>

(Note: subscript $f$ denotes fatigue loads)
2.3.3. Boundary conditions

The loads given in Tables 5 and 6 are applied as concentrated loads on the tower top for static analysis and fatigue analysis, respectively. The wind turbine weight with a value of 480,076kg (LaNier, 2005) is taken into account by adding a point mass on the tower top. For ultimate load case, both gravity loads due to the weight of the tower itself and the wind loads due to wind passing the tower are taken into account as distributed loads on the tower. Additionally, for both load cases, a fixed boundary condition is applied to the tower bottom to simulate boundary conditions of onshore wind turbines.

3. Genetic algorithm

GA is a search heuristic that mimics the process of natural selection. In GA, a population of individuals (also called candidate solutions) to an optimisation problem is evolved toward better solutions. Each individual has a set of attributes (such as its genotype and chromosomes) which can be altered and mutated. The evolution generally starts with a population of random individuals, and it is an iterative process. The population in each iteration is called a generation, in which the fitness of every individual is evaluated. The fitness is generally the value of the objective function in the optimisation problem being solved. The individuals with higher fitness are stochastically chosen from the current population, and the genome of each individual is modified (such as recombined and mutated) to form a new generation, which is then used in the next iteration. Commonly, the GA terminates when either the current population reaches a satisfactory fitness level or the number of generations reaches the maximum value.

Due to its capability of handling a large number of design variables, GA has been widely applied to optimisation in renewable energy problems. Grady et al. (Grady et al., 2005) applied GA to obtain the optimal placement of wind turbines in the wind farm, maximising production capacity while limiting the number of turbines installed. Lin et al. (Wang et al., 2016a) applied GA to the structural optimisation of vertical-axis wind turbine composite blades, taking account of multiple constraints. The application of GA to the optimisation of aerodynamic shape of wind turbine blades can be found in Refs. (Eke and Onyewudiala, 2010, Polat and Tuncer, 2013). Additionally, GA can also be applied to structural damage detection (Chou and Ghaboussi, 2001) and structural health monitoring of wind turbines (Martinez-Luengo et al., 2016).

GA generally requires a genetic representation of the solution domain and a fitness function to evaluate the solution domain. Each individual can be represented by an array of bits (0 or 1) or other types. Having defined the genetic representation and the fitness function, GA proceeds to initialise a population of candidate solutions and then to improve the population through repeatedly using mutation and crossover operators. The mutation and crossover used in the GA are presented below.
3.1. Mutation

Mutation operator is analogous to biological mutation, and it alters one or more gene values in a chromosome from their initial state. For continuous parameters, the mutation is implemented by a polynomial mutation operation, as illustrated in the following equation.

\[ C = P + (B_u - B_l)\delta \]

where \( C \) is the child, \( P \) is the parent, \( B_u \) is the upper bound of parameters, \( B_l \) is the lower bound of parameters, \( \delta \) is a small variation obtained from a polynomial distribution.

3.2. Crossover

Crossover plays an important role in generating a new generation. Crossover mates (combines) two chromosomes (parents) to generate a new chromosome (offspring). For continuous parameters, crossover operator linearly combines two parent chromosome vectors to generate two new offspring using the following two equations:

\[ C_1 = b \cdot P_i + (1 - b) \cdot P_j \]
\[ C_2 = (1 - b) \cdot P_i + b \cdot P_j \]

where \( C_1 \) and \( C_2 \) are children 1 and 2, respectively; \( b \) is a value between 0 and 1; \( P_i \) and \( P_j \) are parents 1 and 2, respectively.

GA searches for optimal solutions through an iterative procedure, which is summarised below.

1) Define objectives, variables and constraints: The optimisation objectives, design variables and constraints are defined at the first step of GA.
2) Initialise population: Initial population (candidate solutions) is randomly generated in this step.
3) Generate a new population: In this step, a new population is generated through mutation and crossover.
4) Design point update: In this step, GA updates the design points in the new population.
5) Convergence validation: The optimisation converges when having reached the convergence criteria. If the convergence criteria have not yet been reached, the optimisation is not converged and the evolutionary process proceeds to the next step.
6) Stopping criteria validation: If the iteration number exceeds the maximum number of iterations, the optimisation process is then terminated without having reached convergence. Otherwise, it returns to Step 3 to generate a new population.

The above Steps 3 to 6 are repeated until the optimisation has converged or the stopping criterion has been met. Fig. 5 depicts the flowchart of GA.
4. Structural optimisation model of wind turbine towers by coupling FEA and GA

4.1. Objective function

The reduction in wind turbine tower weight is beneficial to reduce the material cost of the tower, achieving successful and economic operation of a wind turbine. Therefore, the minimum tower mass $m_{\text{t}}$ is chosen as the objective function $F_{\text{obj}}$, expressed as:

$$F_{\text{obj}} = \min \, (m_{\text{t}})$$

(7)

4.2. Design variables

Figure 6 presents the schematic of the tower structure. As can be seen from Fig. 6, the tower structure is divided into 16 five-meter-length segments. A linear variation of diameters across the length of the tower is assumed. The top diameter and bottom diameter of the tower and the thickness of each segment are taken as design variables. Thus, 18 design variables are defined, which can be expressed in the following form:

$$X = \left[ x_1 \quad x_2 \quad \cdots \quad x_n \right]^T, \quad n = 18$$

(8)
where \( x_1 \) is the diameter of the tower bottom; \( x_2 \) is the diameter of the tower top; \( x_3 \) to \( x_{18} \) are the thickness of 1\(^{st}\) to 16\(^{th}\) segment, respectively.

![Figure 6. Schematic of tower structure](image)

### 4.3. Constraints

In this study, the structural optimisation of wind turbine towers takes account of six constraint conditions, i.e. deformation, ultimate stress, fatigue, buckling, vibration and design variable constraints.

- **Deformation constraint**

In order to ensure the overall structural stability and to avoid the uncertainties introduced by large deformation, the maximum tower deformation \( d_{\text{max}} \) should not exceed the allowable deformation \( d_{\text{allow}} \). This constraint is given by the following inequality:

\[
d_{\text{max}} \leq d_{\text{allow}}
\]

According to Ref. (Nicholson, 2011), the allowable deformation \( d_{\text{allow}} \) can be determined using the following empirical equation:

\[
d_{\text{allow}} = 1.25 \frac{L}{100}
\]

where \( L \) is the length of the wind turbine tower.

In this study, the tower length \( L \) is 80m, and thus the allowable deformation \( d_{\text{allow}} \) is 1m.
• **Ultimate Stress constraint**

The Von-Mises stress $\sigma$ generated by the loads cannot exceed the allowable stress $\sigma_{\text{allow}}$. This can be expressed in the following inequality forms:

$$\sigma \leq \sigma_{\text{allow}}$$  \hspace{1cm} (11)

The allowable stress $\sigma_{\text{allow}}$ is given by:

$$\sigma_{\text{allow}} = \sigma_y \cdot \gamma_w$$  \hspace{1cm} (12)

where $\sigma_y$ is the yield strength and $\gamma_w$ is the material safety factor.

Yield strength $\sigma_y$ of Steel S355 is 345MPa (EN) for nominal thickness in the range of 16mm and 40mm. The material safety factor $\gamma_w$ is taken as 1.1 according to IEC 61400-1 (Commission, 2005). Thus, the allowable stress $\sigma_{\text{allow}}$ is 314MPa.

• **Fatigue constraint**

Fatigue is particularly important in structures subject to significant cyclic loads. During the operation of the wind turbine, every blade rotation causes stress changes in the wind turbine tower. The rated rotor speed of the WindPACT 5MW wind turbine (the reference wind turbine used in this study) is 12.1rpm (LaNier, 2005), resulting in a loading period of 4.96s. For a service life of 20 years, the number of loading cycles $N_d$ having a period of $T_s$, can be then estimated using:

$$N_d = \frac{20 \text{ [years]} \times 365 \text{ [day/year]} \times 24 \text{ [h/day]} \times 3600 \text{ [s/h]}}{T_s \text{ [s]}}$$  \hspace{1cm} (13)

The fatigue analysis in this study is based on S-N curve method, in which fatigue test results are presented as a plot of stress ($S$) against the number of cycles to failure ($N$). Based on the DEL (Damage Equivalent Load) developed by NREL and detailed in (Freebury and Musial, 2000), computational cost is reduced to an equivalent load case where the number of cycles to failure $N_{\text{del}}$ can be obtained from an equivalent S-N curve. An appropriate S-N curve of slope $m = 4$ and intercept $A = 13.9$ was provided by Ref. (LaNier, 2005) with the DEL loads defined in Table 6 of Section 2.3.3.2.

The minimum fatigue safety ratio $f_{\text{m,req}}$ can be then derived by the ratio of the design stress range $\Delta S_{\text{des}}$ that ensure a design number of cycles $N_d$ over the maximum fatigue stress range $\Delta S_{\text{max}}$ in the structure.

This safety ratio should be greater than the allowable fatigue safety ratio $f_{\text{allow}}$, i.e.:
allows \( r f \geq \text{min} \),

\[(14)\]

\( f_{\text{min}, m} \) is equal to one times the material partial safety factor \( \gamma_{m, f} \) for fatigue. According to IEC 61400-1 (Commission, 2005), the material partial safety factor for fatigue, \( \gamma_{m, f} \), should be not less than 1.1. In this study, 1.1 is chosen for \( \gamma_{m, f} \), and thus \( f_{\text{min}} \) is equal to 1.1.

- **Buckling constraint**

Wind turbine towers generally are thin-wall cylindrical shell structures and are subjected to considerable compressive loads, making them prone to suffer from buckling failure. In order to avoid buckling failure, the load multiplier \( L_w \), which is the ratio of the critical buckling load to the applied load on the tower, should be greater than the allowable minimum load multiplier \( L_{w, \text{min}} \). This constraint can be expressed in the following inequality form:

\[ L_w \geq L_{w, \text{min}} \]  

\[(15)\]

In this study, an value of 1.4 is chosen for the minimum allowable load multiplier \( L_{w, \text{min}} \), according to design standard (GL, 2016).

The buckling analysis module in ANSYS software requires a pre-stress step (static structural analysis) followed by the buckling analysis, and it outputs load multiplier. The critical buckling load is then given by load multiplier times the applied load.

- **Vibration constraint**

In order to avoid the vibration induced by resonance, the natural frequency of the tower should be separated from harmonic vibration associated with rotor rotation, and it usually designed to be within the range of 1P and 3P, which correspond to the frequencies of the rotor. This constraint can be expressed in the following inequality form:

\[ f_{\text{rot}, s} S_f \leq f_{\text{rot}, s} \leq 3 f_{\text{rot}, s} / S_f \]  

\[(16)\]

where \( f_{\text{rot}, s} \) is the frequency associated with rotor rotation; \( f_{\text{rot}, s} \) is the first natural frequency of the tower; \( S_f \) is the safety factor for frequency.

In this study, the rotor rotational speed is 11.2 rpm, and thus the associated frequency \( f_{\text{rot}, s} \) is 0.187 Hz. The frequency safety factor \( S_f \) is taken as 1.05 according to GL standard (Lloyd and Hamburg, 2010).

Substituting \( f_{\text{rot}, s} = 0.187 \text{ Hz} \) and \( S_f = 1.05 \) into Eq. (16) yields:
Design variable constraint

The resultant loads on the wind turbine tower bottom are generally greater than those on the tower top, requiring larger diameter on the tower bottom. Therefore, the diameter of the tower bottom is constrained to be larger than the diameter of tower top, which is expressed as:

\[ x_1 - x_2 \geq 0 \]  

(18)

Moreover, the thicknesses of the tower generally decrease from the tower bottom to tower top. This is ensured by the following constraint:

\[ x_i - x_{i+1} \geq 0 \quad i = 3, 4, \ldots, 17 \]  

(19)

Additionally, each design variable is constrained to vary within a range defined by upper and lower bound. This constraint can be expressed as:

\[ x_i^l \leq x_i \leq x_i^u \quad i = 1, 2, \ldots, 18 \]  

(20)

where \( x_i^l \) and \( x_i^u \) are the lower bound and upper bound of the \( i^{th} \) design variable, respectively.

Table 7 presents the lower and upper bounds of the design variables and the constraint conditions used in the structural optimisation of wind turbine towers.

<table>
<thead>
<tr>
<th>Item</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>Units</th>
<th>Variable definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>5</td>
<td>7</td>
<td>m</td>
<td>Diameter of tower bottom</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>3</td>
<td>6</td>
<td>m</td>
<td>Diameter of tower top</td>
</tr>
<tr>
<td>( x_3 \cdots x_{18} )</td>
<td>0.015</td>
<td>0.040</td>
<td>m</td>
<td>Thickness of tower segments</td>
</tr>
<tr>
<td>( d_{\text{max}} )</td>
<td>-</td>
<td>1</td>
<td>m</td>
<td>Deformation</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>-</td>
<td>314</td>
<td>MPa</td>
<td>Von-Mises stress</td>
</tr>
<tr>
<td>( f_{\text{min}} )</td>
<td>1.1</td>
<td>-</td>
<td>-</td>
<td>Fatigue safety ratio</td>
</tr>
<tr>
<td>( L_{\alpha} )</td>
<td>1.4</td>
<td>-</td>
<td>-</td>
<td>Buckling load multiplier</td>
</tr>
<tr>
<td>( f_{\text{max}} )</td>
<td>0.196</td>
<td>0.534</td>
<td>Hz</td>
<td>Tower natural frequency</td>
</tr>
</tbody>
</table>

4.4. Parameter settings of genetic algorithm
The GA presented in Section 3 is chosen as the optimiser to search for optimal solutions. The main parameters used in GA are listed in Table 8.

Table 8. Main parameter settings of GA

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of initial sampling</td>
<td>Constrained sampling</td>
</tr>
<tr>
<td>Number of initial samples $N_{ini}$</td>
<td>180</td>
</tr>
<tr>
<td>Number of samples per iteration $N_{PerIter}$</td>
<td>50</td>
</tr>
<tr>
<td>Maximum allowable Pareto Percentage [%]</td>
<td>70</td>
</tr>
<tr>
<td>Convergence stability percentage [%]</td>
<td>2</td>
</tr>
<tr>
<td>Maximum number of iterations $N_{MaxIter}$</td>
<td>40</td>
</tr>
<tr>
<td>Crossover probability</td>
<td>0.82</td>
</tr>
<tr>
<td>Mutation probability</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Each parameter in Table 8 is detailed below.

- **Type of initial sampling**
  The initial samples are generally based on constrained sampling algorithm, in which the samples are constrained using design variable constraints defined in Eqs. (18), (19) and (20).

- **Number of initial samples**
  In this study, the number of initial samples $N_{ini}$ is 180, which is 10 times the number of design variables (Phan et al., 2013).

- **Number of samples per iteration**
  In this study, the number of initial samples per iteration $N_{PerIter}$ is 50.

- **Maximum allowable Pareto percentage**
  The Pareto percentage, which is defined as the ratio of the number of desired Pareto points to the number of samples per iteration, is a convergence criterion. The optimisation converges when the Pareto percentage reaches the maximum allowable value (70% in this study).

- **Convergence stability percentage**
Convergence stability percentage is a convergence criterion representing the stability of the population based on its mean and standard deviation. The optimisation converges when this percentage (2% in this study) is reached.

- **Maximum number of iterations**

The maximum number of iterations $N_{\text{MaxIter}}$, which is defined as the maximum possible number of iterations the algorithm executes, is a stop criterion. The iteration stops if this number (40 in this study) is reached. The maximum number of iterations $N_{\text{MaxIter}}$ also provides an idea of the absolute maximum number of evaluations $N_{\text{MaxEval}}$, which can be calculated by:

$$N_{\text{MaxEval}} = N_{\text{Ini}} + N_{\text{PerIter}} \times (N_{\text{MaxIter}} - 1)$$

where $N_{\text{Ini}}$ is the number of initial samples, $N_{\text{PerIter}}$ is the number of samples per iteration.

- **Crossover probability**

Crossover probability, which is the probability of applying a crossover to a design configuration, must be between 0 and 1. A smaller value of crossover probability indicates a more stable population and faster (but less accurate) solution. For example, if the crossover probability is 0, the parents are directly copied to the new population. In this study, a typical value of 0.82 (Gandomkar et al., 2005) is chosen as the probability of crossover.

- **Mutation probability**

Mutation probability, which is the probability of applying a mutation on a design configuration, must be between 0 and 1. A large value of mutation probability indicates a more random algorithm. For example, if the mutation probability is 1, the algorithm becomes a pure random search. In this study, a typical value of 0.01 (Perez et al., 2000) is chosen as the probability of mutation.

### 4.5. Flowchart of the optimisation model

Figure 7 presents the flowchart of the structural optimisation model of wind turbine towers, which combines the parametric FEA model (presented in Section 2) and the GA model (presented in Section 3).
5. Results and discussions

The history of the objective function (mass of the tower) during the optimisation process is depicted in Fig. 8. As can be seen from Fig. 8, the mass of the tower oscillates in the first few iterations and then gradually converges, reaching the best solution with a mass of 259,040 kg at the 11th iteration. A mass reduction of 6.28% is achieved when comparing the optimal tower design against the initial design, which has an initial tower mass of 276,412 kg at 0th iteration.
Figs. 9 to 13 depict the history of the total deformation, maximum von-Mises stress, fatigue safety ratio, buckling load multiplier and first natural frequency of the tower, respectively. The associated allowable values (i.e. upper or lower bounds) are also presented in these figures to strengthen the illustration. As can be seen from Figs. 9 to 13, the fatigue safety ratio is quite close to the allowable values, while other constraint parameters have relatively large margins from the allowable values. This result indicates that the fatigue is dominant in the design in the present case.
Table 9 presents the optimal results of design variables. As can be seen from Table 9, all design variables meet the constraints defined in Eqs. (18), (19) and (20).
The tower deformations, von-Mises stress distributions, buckling analysis results, and first modal frequency of the optimal tower are presented below.

### Deformations

The total deformations of the tower is presented in Fig. 14. As can be seen from Fig. 14, the maximum total deformation is about 0.965m, observed at the tower top. This value is 4% lower than the allowable value of 1m, which indicates the present tower design is stiff enough and not likely to experience large deformations.
The von-Mises stress distributions within the tower structure is presented in Fig. 15. As can be seen from Fig. 15, the maximum von-Mises stress is about 205MPa, and this value is 35% lower than the allowable value of 314MPa, which indicates the present tower design is safe in terms of ultimate stress limit.

The modal analysis is used to calculate the modal frequencies and modal shapes of the tower. In this case, the tower is fixed at the tower bottom and free-vibration (no loads on the tower). Fig. 16 depicts the frequency and modal shape of the first model of the tower. As can be seen from Fig. 16 the first mode frequency is about 0.298 Hz, which is within the desired range of 0.196 Hz and 0.534 Hz.
The buckling analysis results of the tower are depicted in Fig. 17. As can be seen from Fig. 17, the load multiplier is about 3.3, which is 136% higher than the minimum allowable value of 1.4. This indicates the present tower design is not likely to experience buckling failure.

6. Conclusions

In this work, a structural optimisation model for wind turbine towers has been developed by incorporating 1) a parametric FEA (finite element analysis) model, which offers high-fidelity evaluations of the structural performance of the tower; with 2) a GA (genetic algorithm) model, which deals with design variables and finds optimal solutions. The structural optimisation model minimises the mass of the wind turbine tower with multi-criteria constraint conditions. The bottom diameter, top diameter of the tower and the thickness of each tower segment are taken as the design variables. The optimisation model accounts for six constraint conditions, i.e. deformation, ultimate stress, fatigue, buckling, vibration and design variable constraints. The model has been applied to the structural design of a 5MW wind turbine tower. The following conclusions can be drawn from the present study:
• Good agreement (with maximum percentage difference of 2.67%) is achieved in comparison with the
modal analysis results of NREL 5MW wind turbine tower reported in the literature, which confirms
the validity of the present parametric FEA model of wind turbine towers.
• The structural optimisation model of wind turbine towers is capable of accurately and effectively
determine the optimal thickness distributions of wind turbine towers, which significantly improves the
efficiency of structural optimisation of wind turbine towers.
• The mass of the optimal tower is 259.040kg, which is 6.28% lower than the initial design, which
indicates the tower mass can be significantly reduced by using the presentoptimisation model.
• For the optimal tower, the fatigue safety ratio is quite close to the allowable values, while other
constraint parameters (i.e. deformation, maximum von-Mises stress, buckling load multiplier and
frequency) have relatively large margins from the associated allowable values. This indicates the
fatigue is dominant in the design in the present case.

Additionally, the present optimisation model can be used for any practice of structural optimisation of wind
turbine towers, minimising the tower mass with multi-criteria constraint conditions. The proposed
framework is generic in nature and can be applied to a series of related problems, such as the optimisation
of offshore wind turbine foundations with complicated boundary conditions.

References

emissions, energy and costs at the early design stage. Journal of Cleaner Production, 87, 263-274.
Journal of solar energy engineering, 123, 372-381.
Structures, 79, 1335-1353.
Electrotechnical Commission.
Procedia, 24, 194-201.
Springer.
Journal of Research In Engineering, 10.
EN, B. 10025-2 (2004). Hot rolled products of structural steels, Part 2: Technical delivery conditions for non-
alloy structural steels.
FREEBURY, G. & MUSIAL, W. D. 2000. Determining equivalent damage loading for full-scale wind turbine
blade fatigue tests, National Renewable Energy Laboratory.
annealing for optimal DG allocation in distribution networks. CCECE/CCGEI, Saskatoon, 645-648.
algorithms. Renewable energy, 30, 259-270.
HERBERT-ACERO, J. F., PROBST, O., RÉTHORE, F.-E., LARSEN, G. C. & CASTILLO-VILLAR, K. K.
2014. A review of methodological approaches for the design and optimization of wind farms. Energies,
7, 6930-7016.


665 wind turbine for offshore system development. National Renewable Energy Laboratory Golden, CO, 
666 USA.


669 Approaches for Economical Hybrid Steel/Concrete Wind Turbine Towers; June 28, 2002--July 31, 


672 MARTINEZ-LUENGO, M., KOLIOS, A. & WANG, L. 2016. Structural health monitoring of offshore wind 
673 turbines: A review through the Statistical Pattern Recognition Paradigm. Renewable and Sustainable 
674 Energy Reviews, 64, 91-105.

676 of towers supporting utilities. Computers & structures, 82, 1745-1750.


679 Paper, 4938.

681 2013. Design optimization of cold-formed steel portal frames taking into account the effect of building 


687 Media.

689 towers under fatigue loading: Finite element analysis and comparative study. American Journal of 
690 Engineering and Applied Sciences, 8, 489.

690 VAN KUIK, G., PEINKE, J., NIJSSEN, R., LEKOU, D., MANN, J., SORENSEN, J. N., FERREIRA, C., VAN 
691 WINGERDEN, J., SCHLIPF, D. & GEBRAAD, P. 2016. Long-term research challenges in wind 

694 Lancashire.

696 Modelling of A Novel 10MW Vertical-Axis Wind Turbine Rotor Based on Computational Fluid 

699 axis wind turbine composite blades based on finite element analysis and genetic algorithm. Composite 
700 Structures.

702 cross-sectional properties of modern wind turbine composite blades. Renewable Energy, 64, 52-60.

704 for wind turbine blades based on blade element momentum theory and geometrically exact beam 

706 WANG, L., QUANT, R. & KOLIOS, A. 2016b. Fluid structure interaction modelling of horizontal-axis wind 
707 turbine blades based on CFD and FEA. Journal of Wind Engineering and Industrial Aerodynamics, 
708 158, 11-25.

709 ZHAO, X. & MAISSER, P. 2006. Seismic response analysis of wind turbine towers including soil-structure 
711 Dynamics, 220, 53-61.